Seat No. : _______________

N16-108

November-2014

B.Sc., Sem.-V

MAT-304 : Mathematical Programming

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) There are **5** questions.
(2) **All** questions are compulsory.

1. (a) Define a convex set. Show that intersection of two convex subsets of R^n is a convex set. Is the union of two convex subset of R^n is convex set ? **7**

OR

Define convex hull. Prove that convex hull is always convex.

(b) Prove that $S = \{\bar{x} \in R^n : \|\bar{x}\| \leq 1\}$ is a convex set in R^n. Is the set $S_1 = \{\bar{x} \in E^n : \|\bar{x}\| = 1\}$ is convex set ? Justify. **7**

OR

A company making cold drinks has two bottling plants located at towns T_1 and T_2. Each plant produces three drinks A, B and C and their production capacity per day is shown below :

Cold Drink	Plant at	
	T_1	T_2
A	6,000	2,000
B	1,000	2,500
C	3,000	3,000

The marketing department of the company forecasts a demand of 80,000 bottles of A, 22,000 bottles of B and 40,000 bottles of C during the month of November. The operating cost per day of plants at T_1 and T_2 are ₹ 6,000 and ₹ 4,000 respectively. Find (graphically) the number of days for which each plant must be run in November so as to minimize the operating cost while meeting the market demand.

2. (a) Prove that the set of all feasible solutions of an L.P.P. is a closed, convex, bounded below set. **7**

OR

If the set of all feasible solutions (S_F) is a nonempty bounded subset of R^n for a give L.P.P. then Prove that the optimum solution exists at one of the vertices of S_F.

(b) Solve the following L.P.P. by simplex method : **7**

$$\text{Max } Z = 107x_1 + x_2 + 2x_3$$

Subject to condition
$$14x_1 + x_2 - 6x_3 + 3x_4 = 7$$
$$16x_1 + \frac{1}{2}x_2 - 6x_3 \leq 5$$
$$3x_1 - x_2 - x_3 \leq 0$$
$$\text{with } x_1, x_2, x_3, x_4 \geq 0.$$

OR

Solve the following L.P.P. using Big-M method :

$$\text{Max } Z = 3x_1 + 2x_2$$

Subject to condition
$$2x_1 + x_2 \leq 2$$
$$3x_1 + 4x_2 \geq 12$$
$$\text{with } x_1, x_2 \geq 0.$$

3. (a) Define the Dual of L.P.P. Prove that the dual of dual is always primal. **7**

OR

State and prove the Fundamental theorem of Duality.

(b) Solve the following L.P.P. using dual simplex method : **7**

$$\text{Min } Z = 4x_1 + x_2$$

Subject to condition
$$x_1 + x_2 \geq 3$$
$$x_1 - x_2 \geq 2$$
$$\text{with } x_1, x_2 \geq 0.$$

OR

Describe the solution of the following L.P.P. by solving its dual : **7**

$$\text{Max } Z = 3x_1 + 2x_2$$

Subject to condition
$$2x_1 + x_2 \leq 5$$
$$x_1 + x_2 \leq 3$$
$$\text{with } x_1, x_2 \geq 0.$$

4. (a) Prove that for m origins to n destinations balance type transportation problem has m + n – 1 basic variables. **7**

OR

Prove that every transportation problem has a triangular basis.

(b) Solve following transportation problem for minimum transportation cost of MODI's Method : **7**

From ↓ To →	**A**	**B**	**C**	**D**	a_i
I	2	3	11	7	6
II	1	0	6	1	1
III	5	8	15	9	10
b_j	7	5	3	2	17

OR

A marketing manager has 5 salesmen and 5 sales districts. Considering the capabilities of the salesmen and the nature of districts, the marketing manager estimates that sales per month (in hundred rupees) for each salesman in each district would be as follows :

Salesman ↓	**Sales Districts**				
	A	**B**	**C**	**D**	**E**
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

Find the assignment of salesmen to districts that will result in maximum sales.

5. Answer in short. Attempt **all**. **14**

(i) The Dual problem of the L.P.P. : Max $Z = 4x_1 + x_2 + 2x_3$, subject to $2x_1 + 3x_2 + 2x_3 \le 7$, $3x_1 - 2x_2 + 4x_3 = 5$, $x_1, x_2, x_3 \ge 0$ is ……

(ii) The basic feasible solution of $2x_1 + x_2 + 4x_3 = 11$, $3x_1 + x_2 + 5x_3 = 14$ are ……

(iii) Illustrate a L.P. problem of two variable having (α) no solution and (β) infinitely many solution.

(iv) What is the difference between transportation problem and the assignment problem ?

(v) Is $\{(x; y) : a < x < b\}$ the convex subset of R^2 ? Justify.

(vi) Determine an initial basic feasible solution of the following transportation problem using Least Cost method :

From ↓ To →	**D_1**	**D_2**	**D_3**	**D_4**	**a_i**
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
b_j	5	8	7	14	34

(vii) What is the advantage of dual simplex method ?